Enrollment No: ____________________ Exam Seat No: ____________________

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics – III

Subject Code : 4TE03EMT1 **Branch: B. Tech (All)**

Semester : 3 **Date : 25/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) One of the Dirichlet's condition is function *f*(*x*) should be
(A) single valued (B) multi valued (C) real valued (D) none of these

b) Fourier expansion of an even function $f(x)$ in $(-\pi, \pi)$ has
(A) only sine terms (B) only cosine terms
(C) both sine and cosine terms (D) none of these

c) In the Fourier series expansion of $f(x)=|x|$ in $(-\pi, \pi)$, the value of b_n equal to
(A) 0 (B) π (C) 2π (D) $\frac{\pi}{2}$

d) Laplace transform of t^2e^{-3t} is
(A) $\frac{\lceil 2}{(s+3)^2}$ (B) $\frac{3!}{(s+3)^2}$ (C) $\frac{2!}{(s+3)^2}$ (D) $\frac{2!}{(s+3)^3}$

e) Laplace transform of $\frac{\sin t}{t}$ is
(A) $\cot^{-1}\frac{1}{s}$ (B) $\tan^{-1}s$ (C) $\tan^{-1}\frac{1}{s}$ (D) $\sin^{-1}s$

f) $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ is
(A) $\frac{t\cos at}{2a}$ (B) $\frac{t^2\sin at}{2a}$ (C) $\frac{1}{2a^3}(\sin at - at\cos at)$ (D) $\frac{t\sin at}{2a}$

g) $\frac{1}{(D-2)(D-3)(D-4)}\left(e^{4x}+e^{2x}\right)$ equal to
(A) $4x\left(e^{2x}+e^{4x}\right)$ (B) $2\left(e^{2x}+e^{4x}\right)$ (C) $2x\left(e^{2x}+e^{4x}\right)$ (D) none of these

h) The P. I. of $(D^2+1)y=\cosh 3x$ is
(A) $\frac{1}{10}\cosh 3x$ (B) $\frac{1}{10}\sinh 3x$ (C) $\frac{1}{5}\cosh 3x$ (D) none of these

i) The C.F. of the differential equation $(D^3+2D^2+D)=x^2$ is

(A) $y=c_1+(c_2x+c_3)e^{2x}$ (B) $y=c_1+(c_2+c_3x)e^{-x}$ (C) $y=c_1+(c_2x+c_3)e^{x}$
(D) none of these

j) The general solution of the equation $z=px+qy+p^2q^2$ is
(A) $z=ax+by+c$ (B) $z=ax+by+a^2+b^2$ (C) $z=ax+by-a^2b^2$
(D) $z=ax+by+a^2b^2$

k) The solution of the differential equation $(1+y)p+(1+x)q=z$ is
(A) $F\left(x(y-z),\ \frac{x+y+z}{2}\right)=0$ (B) $F\left(y(x-z),\ \frac{x+y+z}{2}\right)=0$
(C) $F\left(z(y-x),\ \frac{x+y+z}{2}\right)=0$ (D) None of these

l) The solution of $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=0$ is
(A) $z=f_1(y+x)+f_1(y-x)$ (B) $z=f_1(y+x)+f_2(y-x)$
(C) $z=f_2(y+x)+f_2(y-x)$ (D) $z=f(x^2-y^2)$

m) The order of convergence in Newton-Raphson method is
(A) 2 (B) 3 (C) 0 (D) None of these

n) The order of convergence in Bisection method is
(A) zero (B) linear (C) quadratic (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Perform the five iteration of the Bisection method to obtain a root of the equation $f(x)=\cos x-xe^{x}$. **(5)**

b) Find the root of the equation $\cos x-3x+1=0$ correct to three decimal positions using False position method. **(5)**

c) Evaluate: $L\left(t\,e^{2t}\cos 3t\right)$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Expand $f(x)$ in Fourier series in the interval $(0,2\pi)$ if **(5)**

$$f(x)=\begin{cases}-\pi, & 0<x<\pi\\ x-\pi, & \pi<x<2\pi\end{cases}$$ and show that $$\sum_{r=0}^{\infty}\frac{1}{(2r+1)^2}=\frac{\pi^2}{8}.$$

b) Obtain Fourier series for the function $f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$ **(5)**

c) Given that one root of the equation $x^3-4x+1=0$ lies between 1 and 2. Find the root correct to 3 significant digits using Secant method. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Using Laplace transform method solve: **(5)**

$$\frac{d^4y}{dt^4}-k^4y=0,\quad y(0)=y'(0)=y''(0)=0,\ y'''(0)=1,\ (k\neq 0)$$

b) Using convolution theorem, evaluate $L^{-1}\left\{\dfrac{s}{\left(s^2+4\right)^2}\right\}$. **(5)**

c) Solve: $pz - qz = z^2 + \left(x+y\right)^2$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Evaluate: $L^{-1}\left[\dfrac{s+2}{(s+3)(s+1)^3}\right]$ **(5)**

b) Solve: $\left(D^2+5D+4\right)y = x^2+7x+9$ **(5)**

c) Solve: $2\dfrac{\partial^2 z}{\partial x^2} - 5\dfrac{\partial^2 z}{\partial x \partial y} + 2\dfrac{\partial^2 z}{\partial y^2} = 5\sin\left(2x+y\right)$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Solve: $\left(D^2-1\right)y = \cosh x \cos x$ **(5)**

b) Show that $x^2 = \dfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \dfrac{\cos nx}{n^2}$ in the interval $-\pi \le x \le \pi$. Hence deduce that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - ..... = \dfrac{\pi^2}{12}$. **(5)**

c) Solve: $L\left(\dfrac{e^{-at} - e^{-bt}}{t}\right)$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Solve by the method of variation of parameters: $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$ **(5)**

b) Solve: $x^3\dfrac{d^3y}{dx^3} + 2x^2\dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$ **(5)**

c) Solve: $\dfrac{\partial^2 z}{\partial x^2} - 4\dfrac{\partial^2 z}{\partial y^2} = \cos 2x \ \cos 2y$ **(4)**

Q-8 **Attempt all questions** **(14)**

a) Using the method of separation of variables, solve $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$, given $u\left(x,\ 0\right) = 6e^{-3x}$ **(7)**

b) The following table gives the variations of periodic current $t = f(t)$ amperes over a period T sec. **(7)**

t (sec) :	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
i (A) :	1.98	1.30	1.05	1.30	– 0.88	– 0.5	1.98

Show, by harmonic analysis, that there is a direct current part of 0.75 amp. in the variable current and obtain the amplitude of the first harmonic.